Ecole d'été de mécanique théorique Théorie du contrôle

Objectives

The objective of the course is to study controllability of open-flows undergoing instabilities through the analysis of

- A model problem : non-parallel, non-linear Ginzburg-Landau equation
- Flow around cylinder for $Re > 47$ (incompressible Navier-Stokes equations)

Of particular interest will be the effect of non-normality in the linearized governing equations (in the Navier-Stokes equations, non-normality is due to the presence of the convection term $u \cdot \nabla u$).

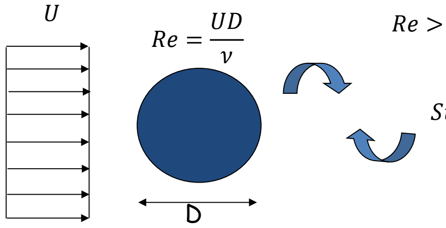
We will try to answer the following questions:

- Can we modify the dynamics of a system?
- Can we suppress instability?
- With **open-loop control**, where should we perform actuation? At what frequency? At what amplitude?
- With **linear closed-loop control**, where should we place the actuator and sensor?

How?

$\Rightarrow$ Build nonlinear amplitude equations taking into account the effect of actuation on the system dynamics $\frac{dA}{dt} = \lambda A + \mu A|A|^2 + \nu E$

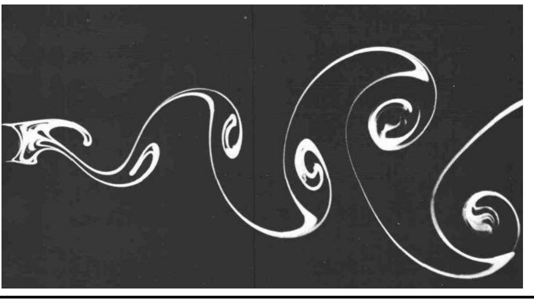
Cylinder flow

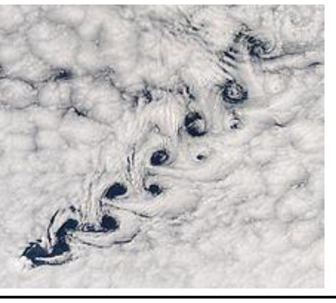


$Re = \frac{UD}{\nu}$

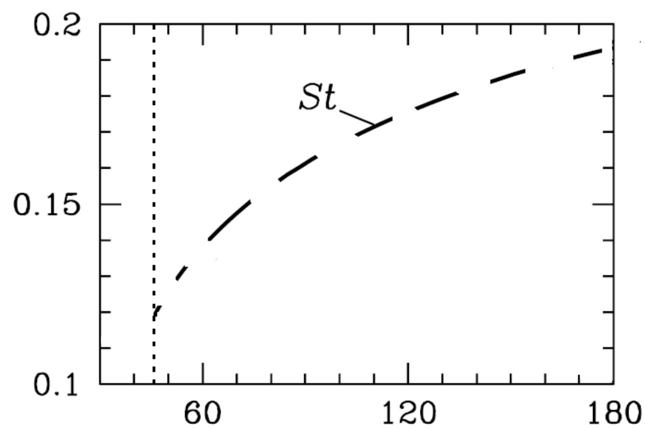
$Re > 47$: appearance of unsteadiness

$St = \frac{fD}{U}$



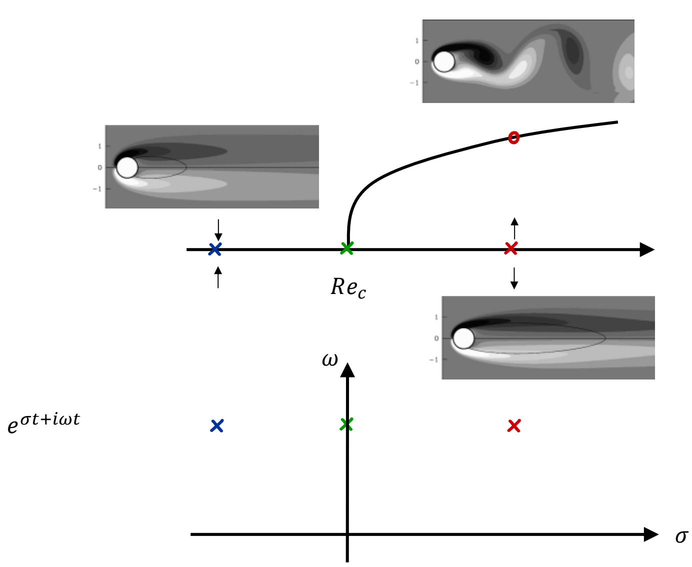


Cylinder flow



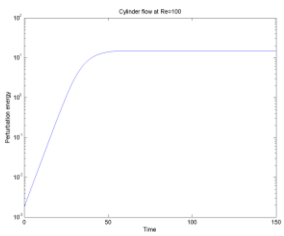
Well-defined peak in frequency spectrum !

Oscillator flows / bifurcation

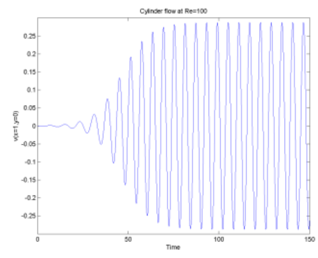


video-dns.mpeg

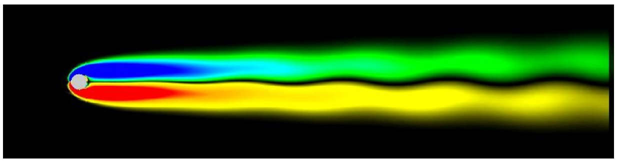
DNS simulation of cylinder flow at Re=100



Energy vs Time



$v(x = 1, y = 0)$ vs Time



Navier-Stokes

Incompressible Navier-Stokes equations:

$$\begin{cases} \partial_t u + u\partial_x u + v\partial_y u = -\partial_x p + \nu\left(\partial_{xx} u + \partial_{yy} u\right) + f \\ \partial_t v + u\partial_x v + v\partial_y v = -\partial_y p + \nu\left(\partial_{xx} v + \partial_{yy} v\right) + g \\ -\partial_x u - \partial_y v = 0 \end{cases}$$

Can be recast into:

$$\mathcal{B}\partial_t w + \frac{1}{2}\mathcal{N}(w, w) + \mathcal{L}w = f$$

where:

$$w = \begin{pmatrix} u \\ p \end{pmatrix} \quad f = \begin{pmatrix} f \\ 0 \end{pmatrix}$$

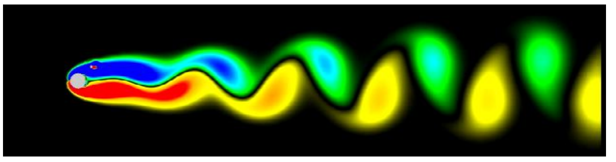
$$\mathcal{B} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$

$$\mathcal{N}(w_1, w_2) = \begin{pmatrix} u_1 \cdot \nabla u_2 + u_2 \cdot \nabla u_1 \\ 0 \end{pmatrix}$$

$$\mathcal{L} = \begin{pmatrix} -\nu\Delta() & \nabla() \\ -\nabla \cdot () & 0 \end{pmatrix}$$

DNS simulation of cylinder flow at Re=100

How does the system respond to forcing?



$$\mathcal{B}\partial_t w + \frac{1}{2}\mathcal{N}(w,w) + \mathcal{L}w = \tilde{E}e^{i\omega_f}f + \text{c.c}$$

Influence of $\omega_f, f, \tilde{E}$?

Open-loop control with time-periodic actuation

Harmonic forcing with synthetic jets Glezer et al. ARFM 2002

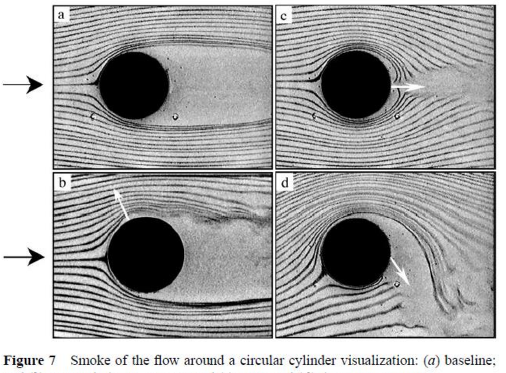


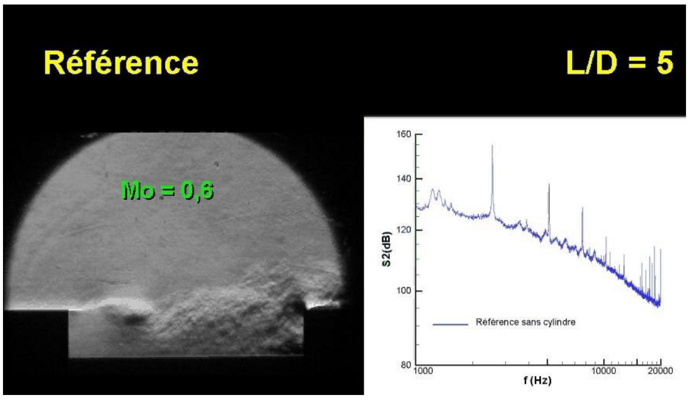
Figure 7 Smoke of the flow around a circular cylinder visualization: (*a*) baseline; and (*b*) actuated: $\phi = 0$, $\gamma = 60°$ and (*c*) 180°, and (*d*) $\phi = 120°$, $\gamma = 180°$.

$$\mathcal{B}\partial_t w + \frac{1}{2}\mathcal{N}(w,w) + \mathcal{L}w = 0$$

$$w = \tilde{E}e^{i\omega_f}g + \text{c.c " on " }\partial\Gamma_c$$

Influence of $\omega_f, f, \tilde{E}$?

Open-loop control with cylinder



Outline of course

1/ Direct and adjoint global modes in open shear-flows (linear behaviour)

2/ Amplitude equations for control (nonlinear behaviour)

Two model problems :
1/ Cylinder flow at Re=100
2/ Non-parallel non-linear Ginzburg-Landau equations

Difficulty : Non-normality of linearized operator due to convection $u \cdot \nabla u$
Operator:

$$u' \to u' \cdot \nabla u_b + u_b \cdot \nabla u'$$

is not symmetric.

Ginzburg-Landau

Forced nonparallel, nonlinear Ginzburg-Landau equation:

$$\partial_t w + U\partial_x w + w|w|^2 = \mu(x)w + \gamma\partial_{xx}w + f(x,t)$$
$$\mu(x) := i\omega_0 + \mu_0 - \underbrace{\gamma\chi^4}_{=\mu_2} x^2$$
$$|w| \to 0 \text{ as } x \to \pm\infty$$

and U, γ, ω_0, μ_0, μ_2 are positive real constant, $f(x,t)$ a “weak” forcing

Ginzburg-Landau

Advection:

$$\partial_t w + U\partial_x w = 0$$

Diffusion:

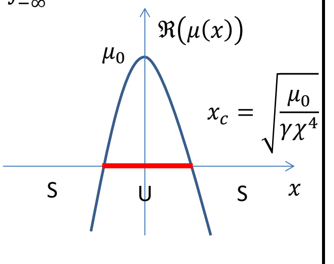
$$\partial_t w = \gamma\partial_{xx}w$$

Dissipative non-linearity:

$$\partial_t w + w|w|^2 = 0$$
$$\partial_t\left(\frac{|w|^2}{2}\right) + |w|^4 = 0 \Rightarrow \frac{\mathrm{d}}{\mathrm{dt}}\left(\int_{-\infty}^{+\infty}\frac{|w|^2}{2}dx\right) = -\int_{-\infty}^{+\infty}|w|^4 dx < 0$$

Localized in-space instability term:

$$\partial_t w = \mu(x)w$$
$$\mu(x) := i\omega_0 + \mu_0 - \gamma\chi^4 x^2$$



Forcing:

$$\partial_t w = f(x,t)$$